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Useful geodetic measurements with radio interferometers?

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Gold pointed out in 1967 that as some quasars have components smaller than 0.01″, radio interferometers might allow distances on Earth of the order of 10^4 km to be measured to a precision of a few centimetres. He envisaged radio interferometers employing video tape recorders and atomic time standards. The difficulties and probable costs of such measurements are reviewed.

INTRODUCTION

By 1967 direct measurements with radio interferometers having a baseline of 129 km (Palmer *et al.* 1967) had shown that some quasars had angular sizes smaller than 0.03″ (seconds of arc). Indirect arguments suggested that some quasars were much smaller still, and Gold (1967) predicted that they would still give detectable correlation even when interferometers used baselines comparable with the diameter of the Earth. He also pointed out that, in principle, these measurements would determine the linear separation of radio telescopes in different continents with sufficient accuracy to be of interest to geophysicists, and yet refer the measurements to an extragalactic frame of reference. No such measurements have been published yet. The proposed methods and their difficulties are discussed in the next section, and some likely developments of future work are mentioned in §3.

RADIO INTERFEROMETERS

Almost all the radio interferometers constructed before 1967 used radio frequency cables or radio links to make the receivers coherent, and to bring the broadband noise signals together for correlation. The proposed new interferometers would use two independent atomic time standards to replace the c.w. coherence links and would replace the broadband 'noise' channels by video tape recorders. The tapes would be recorded simultaneously at two widely separated observatories, and later brought to a central computer for correlation.

The basic geometry of an interferometer is shown in figure 1. If we assume that the electrical paths from the telescopes to the correlator have equal lengths, then if full coherence is to be maintained a compensating delay

$$\tau = (D/c)\cos\theta \tag{1}$$

must be inserted to balance the air path delay in one arm of the instrument. (D is the length of the baseline, c the velocity of light, and θ the angle between the baseline and the normal to the incident wavefront.) θ can be expressed in celestial coordinates to give

$$\tau = -(D/c)[\sin\delta_b \sin\delta_s + \cos\delta_b \cos\delta_s \cos(L_s - L_b)], \tag{2}$$

where $\delta_b\, L_b$ are the declination and hour angle of the intersection between the baseline projected and the celestial sphere, and $\delta_s\, L_s$ are the celestial coordinates of the radio source being observed.

As θ changes with time the interferometer record will show a sinusoidal 'fringe pattern' of period T where

$$\frac{1}{T} = \frac{-\omega}{2\pi c}\left[(D\cos\delta_{\mathrm{b}})\cos\delta_{\mathrm{s}}\sin(L_{\mathrm{s}}-L_{\mathrm{b}})\right]\frac{\mathrm{d}L_{\mathrm{s}}}{\mathrm{d}t}. \tag{3}$$

and ω is the angular frequency of observation.

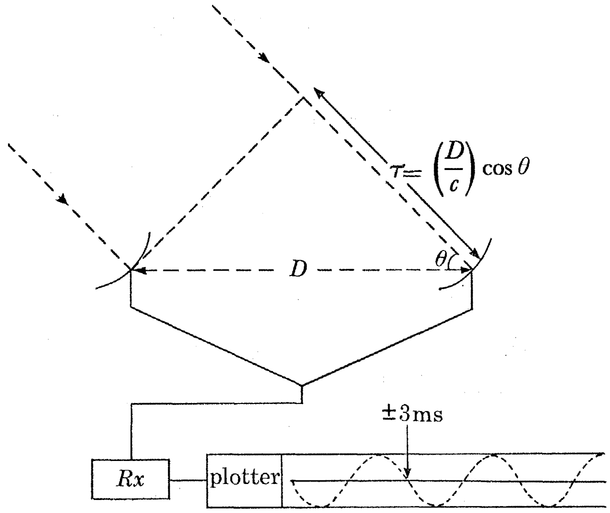

FIGURE 1. The basic geometry of a radio interferometer.

After observations of two or more quasars having known values of δ_{s} and L_{s}, L_{b} can be found and $D\cos\delta_{\mathrm{b}}$ can be calculated from equation (3). If frequency drifts of δf arise between the atomic standards, of output frequency f, there will be errors in ω of $\delta\omega = \omega\,\delta f/f$. When these frequency drifts are the predominant source of error, it can easily be shown that the resulting error δD in D

$$= \frac{c}{\omega_{\mathrm{E}}}\frac{\mathrm{d}f}{f} = 5\text{ cm } (\delta\tau \equiv 0.15\text{ ns}), \tag{4}$$

where $\omega_{\mathrm{E}} = 1/1.4\times10^4$ rad/s is the angular velocity of the Earth. For a baseline of 5000 km this corresponds to a fractional error of 1 part in 10^8, so errors in θ must not exceed 0.01″, and the effective centre frequency of the recorded bandwidth of perhaps 10^6 Hz must be known to better than 1 part in 10^8 of the frequency of observation.

So long as the above assumptions are valid, δD does not vary with observing frequency, integration time or length of baseline. The atmospheres and ionospheres above the observatories introduce corrections which have been fully investigated by Mathur, Grossi & Pearlman (1970). They show that for observations at 1660 MHz, the corrections varied $\pm$ 15 m. With time of day and prevailing conditions. Most of this correction comes from the ionosphere, and therefore falls off as $1/f^2$, but even at the highest frequencies the troposphere contributes a slightly variable correction of order 2 m, according to elevation.

In principle equation (2) can be used directly to calculate the delay τ and hence the length of the baseline. To achieve an accuracy of 5 cm the delay error must not exceed 0.15 ns, and hence the effective bandwidth must be of order 6 GHz. Since video tape recorders are limited to bandwidths of a few million Hertz, Rogers (1970) has suggested the use of a 'sampling' technique to widen the effective r.f. bandwidth. He has so far achieved an increase by a factor of about 8, but this bandwidth is still a factor of order 10^2 smaller than that required.

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Use of equations (2) or (3) with a single baseline implies that the radio source coordinates and atmospheric corrections etc., will be determined separately to the required accuracy. (At present some quasar positions are known with errors ⩽ 0.3″.) Ardoom (1972) has pointed out that a net of four stations used simultaneously, each with radio telescope and hydrogen maser frequency standard, would give sufficient information for the source positions, frequency and path length corrections to be calculated as well as the baseline lengths and directions. This proposal follows similar ones for satellite and lunar reflexion observations. The demands on individual measurements, for a final accuracy of 5 cm, have not been calculated.

Conclusion

Radio observation of remote quasars should permit changes of length, ⩾ 5 cm to be determined for any baseline on Earth. Changes in baselines shorter than 100 km could probably be determined with errors ⩽ 1 cm. If a net of four stations were used simultaneously absolute values for distances of order 5000 km could be referred to an extragalactic frame of reference.

The expensive equipment involved, in addition to four radio telescopes larger than ≃ 50 m in diameter, is four maser frequency standards (≃ £30000 each) and four low-noise receivers and video tape recorders of similar cost.

References (Palmer & Anderson)

Ardoom, L. 1972 Preprint from *Journées Luxembourgeoises Geodynamique*, January 1972.
Gold, T. 1967 *Science, N.Y.* **157**, 302.
Mathur, N. C., Grossi, M. D. & Pearlman, M. R. 1970 *Radio Sci.* **5**, 1253.
Palmer, H. P., Rowson, B., Anderson, B., Donaldson, W., Miley, G. K., Gent, H., Adgie, R. L., Slee, O. B. & Crowther, J. H. 1967 *Nature, Lond.* **213**, 789.
Rogers, E. E. 1970 *Radio Sci.* **5**, 1239.

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